Physics Notes

BY

Er. Lalit Sharma
B.Tech (Electrical)
Ex. Lecturer Govt. Engg. College Bathinda
Physics Faculty Ranker's Point, Bathinda

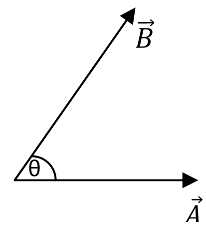
Arun Garg
M.Sc. Physics
Gold Medalist
Physics Faculty Ranker's Point, Bathinda

Class:10+1
Unit: IV
Topic: Work, Power, Energy

SYLLABUS: UNIT-IV

Scalar product of vectors, Work done by a constant force and a variable force; kinetic energy, work-energy theorem, power, Notion of potential energy, potential energy of a spring, conservative forces; conservation of mechanical energy (kinetic and potential energies); non-conservative forces; elastic and inelastic collisions in one and two dimensions.

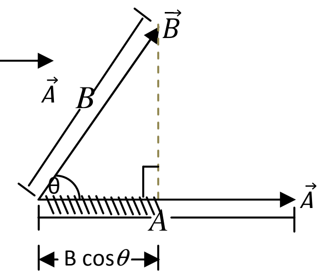
Q.1. Dot product of two vectors? Geometrical interpretation? Properties?



Ans.a) **Dot Product**:- $\underbrace{\vec{A}\,.\,\vec{B}}_{\text{Scalar}} = \underbrace{A.B.\,Cos\theta}_{\text{Scalar}}$

Dot product of two vectors $\vec{A}$ and $\vec{B}$ is equal to the product of magnitudes of $\vec{A}$ and $\vec{B}$ and the cosine of angle between two vectors.

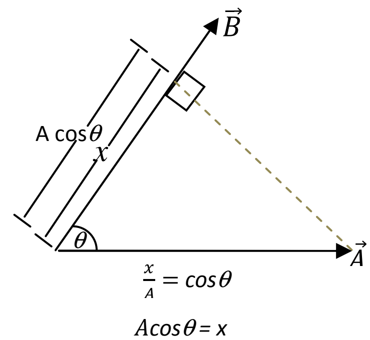
b) **Geometrical Interpretation**:-



$\vec{A}\,.\,\vec{B}$ = A.B. Cosθ = A (B Cosθ)

= A [Component of B in direction of $\vec{A}$]

OR

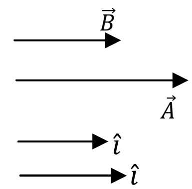


$\vec{A}\,.\,\vec{B}$ = A.B. Cosθ = B (A Cosθ)

= B [Component of A in direction of $\vec{B}$]

c) **Some special Cases**:-

Case I:- $\vec{A}$ and $\vec{B}$ are parallel



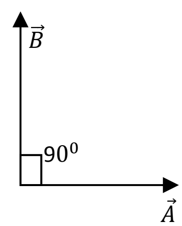
$\vec{A}\,.\,\vec{B}$ = A.B. Cos0^0 = A. B

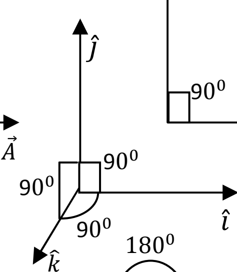
Example:- $\hat{\imath}\,.\,\hat{\imath}$ = 1 . 1 Cos0^0 = 1

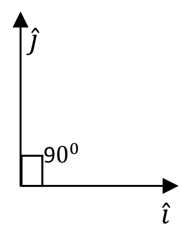
$\hat{\jmath}\,.\,\hat{\jmath}$ = 1 . 1 Cos0^0 = 1

$\hat{k}\,.\,\hat{k}$ = 1 . 1 Cos0^0 = 1

Case II:- $\vec{A} \perp \vec{B}$







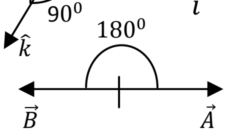
$\vec{A}\,.\,\vec{B}$ = A.B. Cos 90^0 = A. B (0) = 0

Example:- $\hat{\imath}\,.\,\hat{\jmath}$ = 1 . 1 Cos 90^0 = 1.1.(0) = 0

$\hat{\jmath}\,.\,\hat{k}$ = 1 . 1 Cos 90^0 = 1.1. 0 = 0

$\hat{k}\,.\,\hat{\imath}$ = 1 . 1 Cos 90^0 = 0

Case III:- $\vec{A}$ is anti parallel to $\vec{B}$



$\vec{A}\,.\,\vec{B}$ = A.B. Cos 180^0 = A. B (-1) = -A.B

d) **Properties**:-

1. $\vec{A}\,.\,\vec{B} = \vec{B}\,.\,\vec{A}$ $\vec{A}\,.\,\vec{B}$ = A.B. Cosθ; $\vec{B}.\vec{A}$ = B.A. Cosθ
2. $\vec{A}\,.\,(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$
3. $\vec{A}\,.\,\vec{A}$ = A.A. Cos0^0 = A.A.(1)
4. $\vec{A}\,.\,(3\vec{B}) = 3(\vec{A}\,.\,\vec{B})$ = 3[A.B. Cosθ]

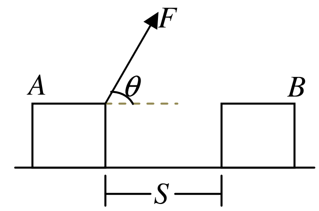
Example:-

1. W $= \vec{F}\,.\,\vec{S}$

2. $\frac{W}{t} = \frac{\vec{F}.\vec{S}}{t}$

⇒ P $= \vec{F}\,.\,\vec{v}$

Q.2. Define Work? Units? Dimensions?



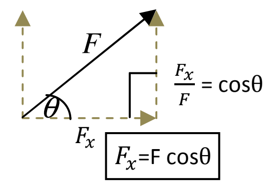
Ans. **<u>Work</u>**:-

W $= \vec{F} \cdot \vec{S}$

= F.S. Cosθ

= S(F Cosθ)

= [Component of force in direction of $\vec{S}$]



Work is said to be done by a force when a body is displaced actually through some distance in the direction of applied force.

<u>Units</u>:-

SI Unit = Joule

Cgs Unit = erg

1 Joule = 10^7 erg

Work = (1N) x (1m) = (10^5 dyne) (100cm) = 10^7 erg

<u>Dimensions</u>:-

[W] = [F] [S]

= $[M^1L^1T^{-2}]$ [L]

[W] = $[M^1L^{12}T^{-2}]$

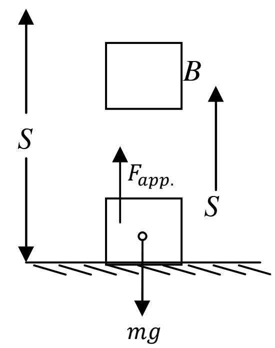
Q.3. Explain:-

a) +ve work

b) –ve work

c) Zero work

Ans.a) **<u>+ve Work</u>**:-



Work done, W $= \vec{F} \cdot \vec{S}$

$= F.S. \cos\theta$

F,S → +ve

W is +ve if Cosθ is +ve

W is –ve if Cosθ is –ve

∴ W is +ve if $0^0 \leq \theta \leq 90^0$

<u>Example</u>:- Work done by applied force, $F_{app.}$

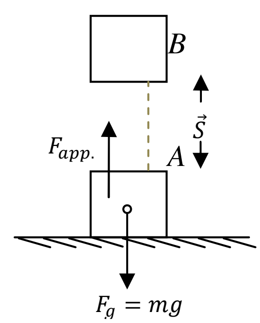
$W_{app.} = \vec{F}_{app} \cdot \vec{S}$

$= F_{app.} \cdot S \cdot \cos\theta$

$= F_{app.} \cdot S \cdot \cos 0^0$

$= F_{app.} \cdot S$

b) **<u>–ve Work</u>**:- W $= \vec{F} \cdot \vec{S}$



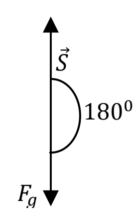
$= F.S. \cos\theta$

F,S → +ve

W is –ve if Cosθ is –ve

∴ W is -ve if $0^0 \leq \theta \leq 180^0$

<u>Example</u>:- $W_{gravity} = \vec{F}_g \cdot \vec{S}$



$= F_g \cdot S \cdot \cos\theta$

$= F_g \cdot S \cdot \cos 180^0$

$= F_g (S)(-1)$

$= -F_g \cdot S$

= (-10) (2) [say]

= -20 J [say]

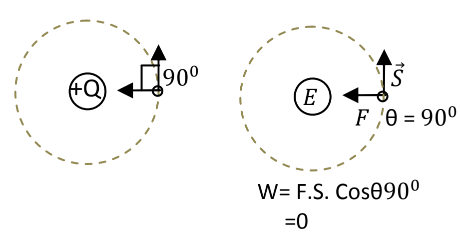
c) **<u>Zero Work</u>**:- W = F.S. Cosθ

Work is 'ZERO" if F = 0

OR S = 0

OR Cosθ = 0 i.e. $\theta = 90^0$

<u>Example</u>:- W $= \vec{F}_C \cdot \vec{S}$



$= F_C \cdot S \cos 90^0$

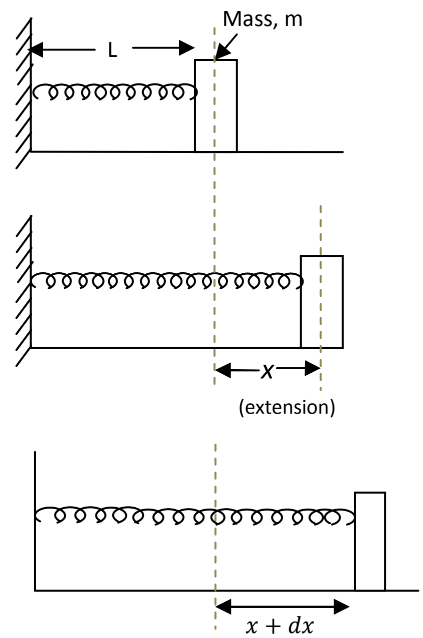
= 0

- Force of satellite on earth, $\theta = 90^0$

Q.4. For a spring, plot

a) $F_{restoring}$ v/s extension graph

b) $Energy$ v/s extension graph



Ans. L → Natural length of a massless spring

K → Spring constant, $F_{restoring}$ per unit length (say $\frac{10N}{cm}$)

$F_{restoring}$ α extension (say x)

$F_{restoring}$ = -k . x.

Spring constant

If $F_{restoring}$ is +ve, x is –ve

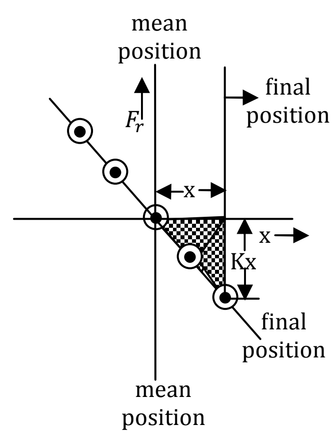
If $F_{restoring}$ is –ve, x is +ve

a) Y = -10x

x	0	1	-1	2	-2
y	0	-10	10	-20	20

Graph – straight line passing through centre

Slope = -K



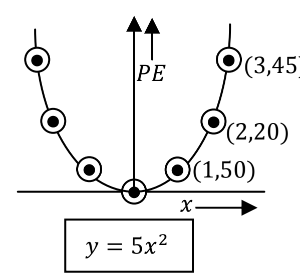
b) **Method I**:-

Work done = Area under F – x graph

$= \frac{1}{2}$ x Base x Height

$= \frac{1}{2}$ x X x KX

E.P. E_x $= \frac{1}{2} kx^2$



Method II:-

Work $= \int F.dx$

$= \int (Kx).dx$

$=$ K $\int x^1 .dx$

$=$ K $\left|\frac{x^{1+1}}{1+1}\right| \begin{matrix} x = x \\ x = 0 \end{matrix}$

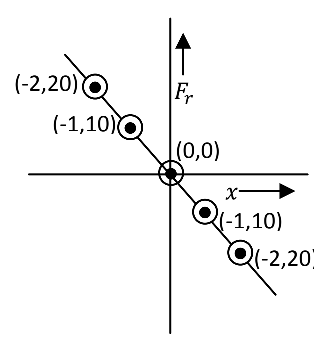
$= \frac{1}{2}$ K $(x^2 - 0^2)$

E.P.E_x $= \frac{1}{2}$ K x^2

Graph → Parabola

Note: Whether x +ve or -ve, Energy is stored

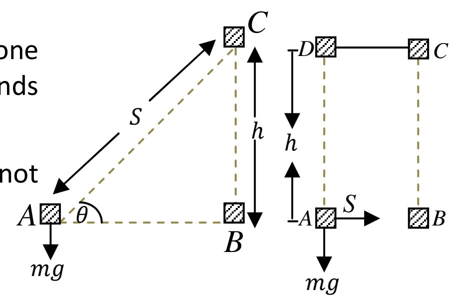
$\int x^n \, dx = \frac{x^{n+1}}{n+1}$



Q.5. What are "Conservative" and "Non-Conservative" forces? Discuss properties? With examples.

Ans. **<u>Conservative Force</u>**:-

Conservative Force is force in which work done does not depend on the path followed. It only depends on initial and final positions of the object.



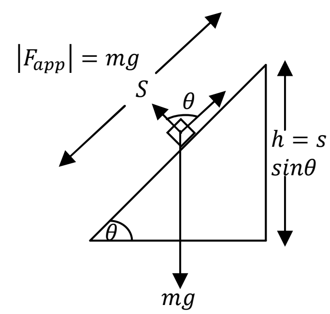
<u>Example</u>:- Work done in this case is stored and not dissipated

W_{ABC} = $W_{AB} + W_{BC}$

= (mg)(AB) $\cos 90^0$ + (mg)(h) $\cos 0^0$

= 0 + mgh

$$W_{ABC} = mgh \qquad \text{----------------- (1)}$$



W_{AC} = $|(mg)(s)|$ cos (90+0)

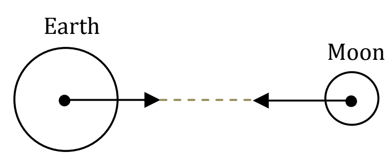
W_{AC} = mg s sinθ

$$W_{AC} = mgh \qquad \text{----------------- (2)}$$

So, $W_{ABC} = W_{AC}$ i.e. work done on path ABC is same as that of work done on path AC

<u>Other Examples</u>:-

1. Spring Force
2. Electrostatics Force
3. Magnetic Force

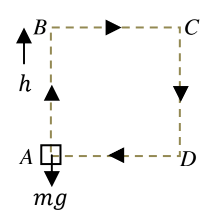


Last two forces (2 & 3) are central forces

Central forces are forces acting along the line joining the centres of two objects/charges etc.

<u>Properties</u>:-

1. Depends only upon initial and final position.
2. Does not depend upon the nature of the path followed.
3. $W_{gravity}$ ABCD = $W_{AB} + W_{BC} + W_{CD} + W_{DA}$



= (mg)(h) $\cos 180^0$+ 0 +(mg)(h) $\cos 0^0$ +0

= mgh (-1) + 0 + mgh(1) +0

= -mgh + mgh

= 0

∴ Work done through a round trip is always zero.

Non Conservative:-

Non Conservative force is force in which work done depends upon the path followed.

Example:-

If a body is moved from position *A* to another position *B* on a rough table, work done against frictional force shall depend upon the length of path between *A* & *B* and not only on the positions of *A* & *B*.

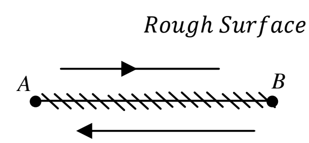
Other Example:-

Induction Force in a cyclotron. The charged particle returns to its initial position with more Kinetic Energy than what it had originally.

Properties:-

1. Does not depend only upon initial & final position of body.
2. Depends upon the nature of the path followed.
3. Work done against frictional force in moving a body from

A to B = 100J (say)



Then work done to bring back this body from

B to A = 100J

This is because friction is self adjusting

$W_{AB} + W_{BA}$ = 100 + 100

= 200J

∴ Hence work done against frictional forces in moving a body over a round trip is not zero.

- Work done is not stored. Work done is lost.

Q.6. Define Power? Relation with force velocity? Units? Dimensions?

Ans. **Power**:-

Power is defined as time rate of change of work or rate of doing work.

i.e. $P = \frac{dw}{dt}$

Relation:- $P = \frac{work}{time}$

$= \frac{dw}{dt}$

$= \vec{F}\left(\frac{d\vec{s}}{dt}\right)$

$P = \vec{F}\,\vec{v}$

Power is scalar

Units:- $\frac{1J}{sec}$ i.e. 1 watt

1 H.P = 746 watt

Dimensions:- $[P] = \frac{[W]}{[T]}$

$= \frac{[F][S]}{[T]}$

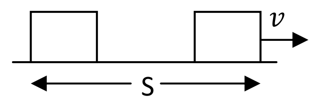
$= \frac{[M^1L^1T^{-2}][L]}{T}$

$$[P] = [ML^2T^{-3}]$$

Q.7. What is Kinetic Energy? Prove K.E. = $\frac{1}{2} m v^2$?

Ans. **<u>Kinetic Energy</u>**:-

Kinetic energy of a body is the energy possessed by the body by virtue of its motion.



K.E. = Work done

$v^2 - u^2 = 2as$

$v^2 - 0 = 2as$

$a = \frac{v^2}{2s}$

As F = m a

$F = m\left(\frac{v^2}{2s}\right)$

Work done, W = Force x Distance

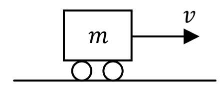
$W = m\frac{v^2}{2s} \times S$

$W = K.E. = \frac{1}{2} m v^2$

Hence Proved.

Q.8. **a) Fine relation between Kinetic Energy and Linear Momentum.**
b) Plot:-
I) *K-m* graph when *þ* is constant
Ii) *þ-m* graph when *K.E.* is constant
Iii) *K-þ* graph when *m* is constant

Ans.a) **<u>K.E. þ relation</u>**:-



K.E. $.= \frac{1}{2} m\, v^2$ --------------- (1)

Þ $= m\, v$ --------------- (2)

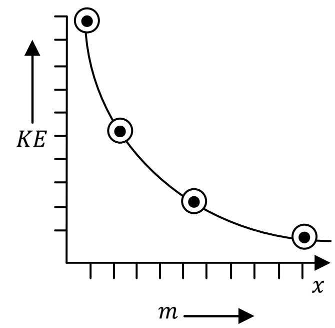
From (2) $v = \frac{\text{Þ}}{m}$

Put in eq.(1)

K.E. $.= \frac{1}{2} m \left(\frac{\text{Þ}}{m}\right)$

$$\text{K.E.} = \frac{\text{Þ}^2}{2m}$$

b)I) **<u>K-m Graph</u>**:-



K.E. $= \frac{\text{Þ}^2}{2m}$

K.E. $= \frac{constant}{m}$ [Þ = constant]

Y $= \frac{constant}{m}$ Say Y $= \frac{10}{x}$

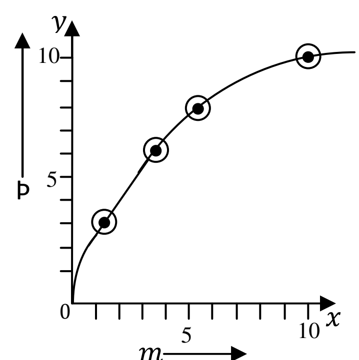
x	1	2	5	10
y	10	5	2	1

As *x* increases *y* decreases

∴ y α x

K.E. α m [Þ → constant]

II) ***<u>þ-m</u>* <u>graph</u>**:- [K.E. → constant]

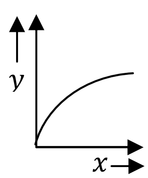


K.E. $= \frac{\text{Þ}^2}{2m}$

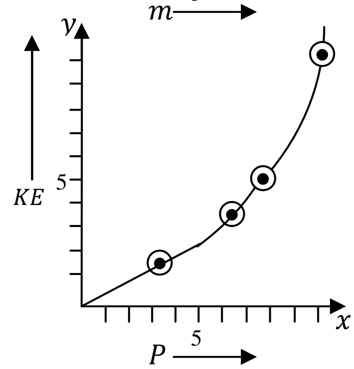
Constant $= \frac{y^2}{x}$

Say $10x = y^2$

y	10	3.1	7.1	6.3
x	10	1	5	4



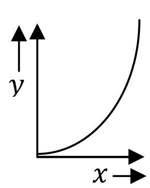
III) ***<u>K-þ</u>* <u>graph</u>**:- [m → constant]



K.E. $= \frac{\text{Þ}^2}{2m}$

K.E. $= \frac{\text{Þ}^2}{constant}$ ∴ $y = \frac{x^2}{constant}$ say $10y = x^2$

x	10	3.1	7.1	6.3
y	10	1	5	4

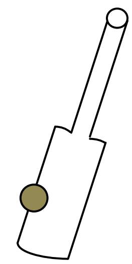


Q.9. Discuss elastic and inelastic collision.

Ans. **<u>Collision</u>**:-

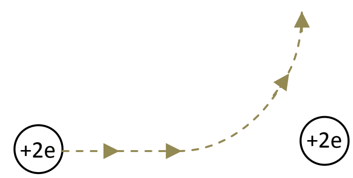
An isolated event in which two or more colliding bodies exert relatively strong forces on each other for relatively short time.

Contact type collision is one in which two bodies come in physical contact during collision.



<u>Example</u>:- Ball hitting the bat.

Non-contact type collision is one in which two bodies exert force on each other without, coming in physical contact.



<u>Example</u>:- α – particle getting deflected due to nucleus.

<u>Elastic Collision</u>:-

Collision in which there is no loss of kinetic energy.

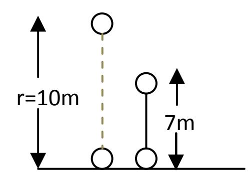
<u>Example</u>:- Collision between two atoms.

<u>Characteristics</u>:-

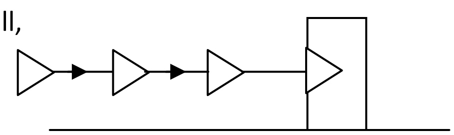
1. Linear momentum is conserved.
2. Total energy of the system is conserved.
3. Kinetic Energy before collision = Kinetic Energy is after collision
4. Forces involved must be conservative forces.

<u>Inelastic Collision</u>:-

Collision in which Kinetic Energy before collision is not equal to Kinetic Energy after collision i.e. some Kinetic Energy is lost.



<u>Example 1</u>:- Inelastic ball bouncing on ground



<u>Example 2</u>:- Perfectly inelastic bullet fired in wall, bullet & wall becomes one

<u>Characteristics:</u>-

1. Linear momentum is conserved.
2. Total energy of the system is conserved.
3. Kinetic Energy before collision ≠ Kinetic Energy after collision. A part of Kinetic Energy is converted into some other form of energy
   <u>Example</u>:- Heat & Sound Energy, etc.
4. Some on all the forces involved in inelastic collision may be non-conservative in nature.

Q.10 Define coefficient of restitution or coefficient of resilience. Write significance for its extreme values.

Ans. e $= \dfrac{rel.velocity\ of\ separation\ (after\ collision)}{rel.velocity\ of\ approach\ (before\ collision)}$

e $= \dfrac{v_2 - v_1}{u_1 - u_2}$

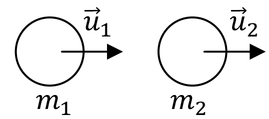
$0 \le e \le 1$

When e = 0 perfectly inelastic
e = 1 elastic

Q.11. Derive an expression for final velocities in an "Elastic Collision". Discuss special cases.

Ans.1. Law of conservation of momentum.

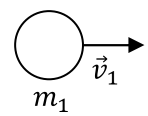
Total momentum before collision = Total momentum after collision



$m_1 v_1 + m_2 v_2 = m_1 v_1 + m_2 v_2$ --------------- (1)

2. Kinetic Energy before collision = Kinetic Energy after collision

$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ ---------- (2)



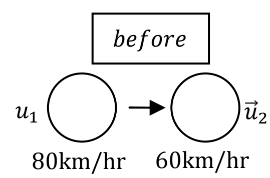
On solving these two equations, we get

$v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] . u_1 + \left[\frac{2m_2}{m_1 + m_2}\right] u_2$

$v_2 = \left[\frac{2m_1}{m_1 + m_2}\right] . u_1 + \left[\frac{m_2 - m_1}{m_1 + m_2}\right] u_2$

<u>Special Cases</u>:-

<u>Case I</u>:- $m_1 = m_2$



$v_1 = \left|\frac{m_1 - m_2}{m_1 + m_2}\right| v_1 + \left|\frac{2m_2}{m_1 + m_2}\right| u_2$

$= \left|\frac{m - m}{m + m}\right| v_1 + \left|\frac{2m}{m + m}\right| u_2$

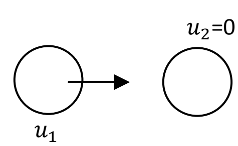
$v_2 = \left|\frac{2m}{m + m}\right| v_1 + \left|\frac{m - m}{m + m}\right| u_2$

$v_1 = v_2$

$v_2 = v_1$

<u>Conclusion</u>:- Two particles exchange their velocities.

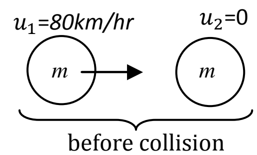
<u>Case II</u>:- When m_2 is at rest $u_2 = 0$



$v_1 = \left|\frac{m_1 - m_2}{m_1 + m_2}\right| u_1 + 0$

$v_2 = \left|\frac{2m_2}{m_1 + m_2}\right| u_1 + 0$

2(a) $m_1 = m_2$



$$v_1 = \left|\frac{m-m}{m+m}\right| u_1 = 0$$

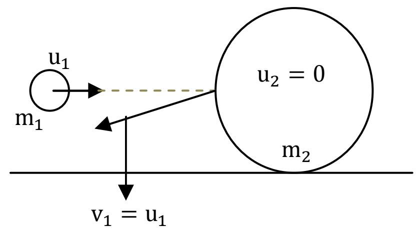
$$v_2 = \left|\frac{2m}{m+m}\right| u_1 + u_1$$

$$\boxed{\begin{array}{l} v_1 = 0 \\ v_2 = u_1 \end{array}}$$

2(b) When body at rest *B* is very heavy, $m_2 >> m_1$

$$v_1 = \left|\frac{m_1-m_2}{m_1+m_2}\right| u_1 + 0$$

$$= \left|\frac{\frac{m_1}{m_2}-1}{\frac{m_1}{m_2}+1}\right| u_1 = \left(\frac{-1}{+1}\right) u_1$$



$$\simeq - u_1$$

$$\boxed{v_1 = - u_1}$$

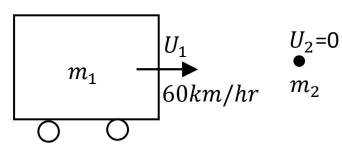
$v_2 \simeq 0$ Approximately equal to zero

First body bounces back with almost same speed and 2nd body remains at rest.

2(c) When body *B* at rest is very very small, $m_1 >> m_2$

$$v_1 = \left|\frac{m_1-m_2}{m_1+m_2}\right| u_1 + 0$$

$$= \left|\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right| u_1$$



$$= \left|\frac{1-0}{1+0}\right| u_1$$

$\boxed{v_1 \simeq u_1}$ heavy object moves with same velocity.

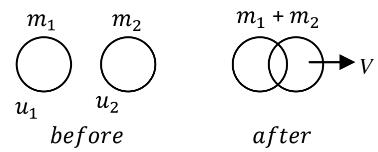
$$v_2 = \left|\frac{2m_1}{m_1+m_2}\right| u_1$$

$$= \left|\frac{2m_1}{1+\frac{m_2}{m_1}}\right| u_1$$

$$\simeq \left|\frac{2}{1+0}\right| u_1$$

$\boxed{v_2 \simeq 2u_1}$ light body at rest will fly away with double velocity.

Q.12 Find expression for
a) Final velocity
b) Loss of Kinetic Energy
In perfectly inelastic collision in one dimension.



Ans.a) **Final Velocity, *V***:-

As per law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\ V \qquad V \rightarrow \textit{velocity of combined mass}$$

$$\boxed{V = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}}$$

b) **Loss of Kinetic Energy**:-

$$= K.E_{initial} - K.E_{final}$$

Loss of KE $= \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right] - \left[\frac{1}{2}(m_1 + m_2)\ V^2\right]$

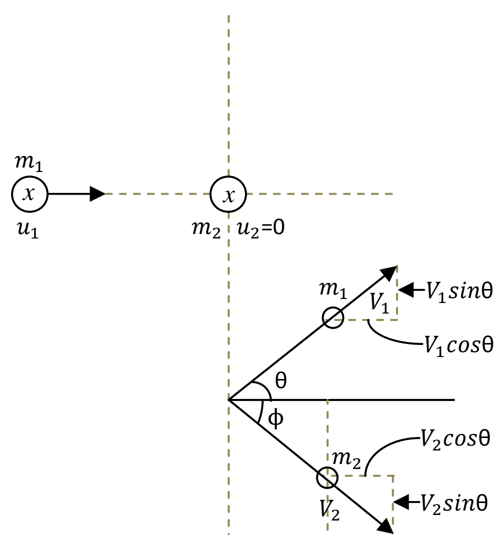
$$KE_{loss} = \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right] - \left[\frac{1}{2}(m_1 + m_2)\right]\left[\frac{(m_1v_1 + m_2v_2)}{m_1 + m_2}\right]^2$$

$$= \frac{1}{2}\left[(m_1v_1^2 + m_2v_2^2) - \frac{(m_1v_1 + m_2v_2)^2}{m_1 + m_2}\right]$$

OR

$$= \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right] - \left[\frac{1}{2}\frac{(m_1v_1 + m_2v_2)^2}{m_1 + m_2}\right]$$

Q.13. Expression for elastic collision in two dimension.



Ans. $\vec{v}_1 = (v_1cos\theta)i + (v_1sin\theta)\ \hat{j}$

$\vec{v}_2 = (v_2cos\theta)i + (v_2sin\theta)\ (-\hat{j}\ )$

Law of conservation of momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Law of conservation of Kinetic Energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$